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CHIRAL PERTURBATION THEORY FOR SU(3) BREAKING IN HEAVY MESON SYSTEMS

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Abstract

The SU(3) breaking effects due to light quark masses on heavy meson masses, decay constants (F_D, F_{D_s}) and the form factor for semileptonic $\overline{B} \to D^{(*)} l \bar{\nu}_l$ transitions are formulated in chiral perturbation theory, using a heavy meson effective Lagrangian and expanding in inverse powers of the heavy meson mass. To leading order in this expansion, the leading chiral logarithms and the required counterterms are determined. At this level, a non-analytic correction to the mass splittings of $\mathcal{O}(p^3)$ appears, similar the the one found in light baryons. The correction to F_{D_s}/F_D is roughly estimated to be of the order of 10% and, therefore, experimentally accessible, while the correction to the form factor is likely to be substantially smaller. We explicitly check that the heavy quark symmetry is preserved by the chiral loops.

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1 Introduction

In addition to their intrinsic significance for the study of electroweak interactions (quark mixing, rare decays, CP-violation), heavy hadrons containing a single heavy quark c or b might also prove to be a useful tool for unveiling new aspects of the strong interactions. The reason for this is the large approximate symmetry available in these systems, which constrains the QCD dynamics and, therefore, substantially simplifies their study. As the mass m_Q of the heavy quark becomes much larger than the characteristic QCD scale (say, m_ρ), in all strong interaction processes where the relevant scale of momenta is much smaller than the heavy quark mass, the heavy quark approximately behaves as a static color source in the rest frame of the hadron, with its spin dynamically decoupled. In this limit, the velocity of the heavy quark and its spin become conserved observables. This results in a superselection rule for the velocity [1, 2], and a spin-flavor symmetry (the Isgur-Wise (I-W) symmetry) [3, 4] which enjoys all bonafide properties of an internal symmetry. In addition, the light quark degrees of freedom in the heavy hadron carry information about the chiral $SU_L(3) \times SU_R(3)$ symmetry of QCD. In particular, chiral symmetry dictates the form of the couplings between the heavy mesons and the Goldstone bosons (π, K, η) resulting from the Nambu-Goldstone nature of its realization (see for instance [5]).

The corrections to the symmetry limit are naturally obtained by expanding in powers of $1/m_Q$ (more precisely in the present context, in powers of $1/M_H$, where M_H is the heavy hadron mass) and by treating the light quark masses, which explicitly break chiral invariance, as a perturbation. The expansion in powers of $1/M_H$ has the virtue of enabling a systematic chiral expansion, at each order in $1/M_H$, where the chiral power counting is in correspondence with an expansion in loops [6], similarly to chiral perturbation theory in light mesons [7]. This only applies to processes involving only one heavy hadron, otherwise, infrared divergences modify the naive chiral counting [8].

$SU(3)$ breaking effects induced by the light quark masses are inherently of low energy character, and therefore, suited to a systematic study within the chiral expansion. Recently, various groups [9, 10, 11] have initiated this field, in which interesting theoretical results are expected to emerge§ . At present, $SU(3)$ breaking is only observed through the mass splittings in D, D^* and B mesons and in charmed baryons. In the future, one also expects observation through other quantities (e.g. decay constants). This requires, however, substantial improvement in strange D meson measurements. As for the decay constants, at present only an upper bound exists for non-strange D mesons: $F_D \leq 200MeV$ ($F_\pi = 93MeV$). For B mesons, observation of $SU(3)$ breaking in form factors (e.g. in $B_{(s)} \to \overline{D}l\nu$ decays) could only be achieved in a B-meson factory. The corrections to $B^0-\overline{B}^0$ and $B_s-\overline{B}_s$ mixing are also of great interest, and have recently been analyzed [10]. Applications in connection with lattice QCD simulations of heavy-light systems can also be envisaged. For instance, finite volume effects in the continuum limit, of relevance in this context, can be unambiguously determined using the chiral expansion [5].

The predictive power of the chiral expansion is limited by the counterterms which must be added at each stage. The counterterms are ordered according to chiral power counting and are required as subtractions to U.V. divergent chiral loops. To overcome this drawback, a sufficient number of measured observables is needed as input. While this is possible in light mesons, it is not yet clear that it can be achieved in heavy mesons.

In this paper, we study the $SU(3)$ breaking corrections, in the limit of infinite heavy quark mass, to the ratio of decay constants $F_{H_s}/F_{H_{u,d}}$ (H denotes a heavy meson), mass splittings, and the Isgur-Wise form factor associated with the charged current in the transitions $B^0 \to D^-$ and $B_s \to \overline{D}_s$. The chiral logarithms and their associated counterterms are determined

§ As we became aware of the works in ref. [9, 10, 11] the present project was approaching its conclusion. Some overlap with some of these references has been unavoidable.

to one-chiral loop order. The presence of non-analytic contributions in the light quark masses ($\alpha\ m_q^{3/2}$) to the mass splittings is noticed.

2 Effective Theory

In this section, we discuss in detail a formulation of an effective theory for heavy (D, B) mesons coupled to the Goldstone bosons of spontaneously broken chiral symmetry. In this formulation, $SU(3)$ breaking effects can be consistently studied in a chiral-loop expansion. This is made possible by simultaneously performing an expansion in powers of the inverse of the heavy meson mass.

The form of the interactions between Goldstone bosons and heavy mesons containing a heavy antiquark and a light quark are determined by the transformation properties of the heavy meson wave functions under chiral $SU_L(3)\times SU_R(3)$. The transformation law is easily found by the method of Coleman, Wess and Zumino [12]. In our case, pseudoscalar and vector heavy mesons appear in triplets under flavor $SU(3)$, and this fixes the transformation law under an arbitrary chiral transformation $g = L \otimes R$ to have the following form:

$$g : H = hH \qquad , \tag{1}$$

where H denotes the heavy meson wave function and h is a 3×3 $SU(3)$ matrix which depends on the octet of Goldstone excitations. Although the explicit form of h will not be needed, it can be determined in the following manner: one defines a 3×3 $SU(3)$ matrix $U(x)$ parametrized by the classical Goldstone fields, and whose transformation law is given by $g : U(x) = L\ U(x)\ R^\dagger$, where on the right hand side ordinary matrix multiplication is meant. By means of $u(x) \equiv \sqrt{U(x)}$, one can determine h in such a way that (1) is a realization of the chiral group. The dependence of h on the Goldstone excitations results from solving the following system of equations:

$$\begin{aligned} L\, u(x) &= u'(x)\, h \\ R\, u^\dagger(x) &= u'^\dagger(x)\, h \end{aligned} \tag{2}$$

In what follows we will use the exponential parametrization for $u(x)$:

$$u(x) = exp\,(-i\frac{\pi_a(x)\lambda^a}{2F_0}) \qquad , \tag{3}$$

where the Goldstone fields $\pi_a(x)$ are real and identified with the light pseudoscalar octet (π, K, η), $F_0 \sim 93MeV$ is the pion decay constant in the chiral limit, and the Gell-Mann hermitian matrices are normalized by $Tr(\lambda^a\lambda^b) = 2\delta^{ab}$.

In order to build the effective Lagrangian invariant under chiral rotations, one needs to introduce a covariant derivative under the transformation law (1). Using the transformation properties of $u(x)$ implied by (2), the explicit form of the covariant derivative is given by:

$$\begin{aligned} \nabla_\mu H &= (\partial_\mu + i\Gamma_\mu)H \\ \Gamma_\mu = \Gamma_\mu^\dagger &= \frac{i}{2}\,(u\partial_\mu u^\dagger + u^\dagger\partial_\mu u) \end{aligned} \tag{4}$$

Besides the covariant derivative, the following hermitian pseudovector must also be considered:

$$\omega_\mu = \frac{i}{2}\,(u\partial_\mu u^\dagger - u^\dagger\partial_\mu u) \tag{5}$$

whose transformation law under a chiral rotation is given by: $g : \omega_\mu = h\omega_\mu h^\dagger$.

Having established the chiral transformation properties for the heavy mesons, we now turn to the important aspects connected with the large mass of the heavy quark. As the mass of the heavy quark tends to infinity, the QCD Lagrangian becomes invariant under a new global symmetry (I-W symmetry) [3], which, in the case of a single heavy quark, corresponds to the operation of independently rotating the spins of the heavy quark and antiquark. This becomes an internal symmetry of QCD, which, for N_h heavy quarks is $SU(2N_h) \times SU(2N_h)$ with one factor referring to quarks and the other to antiquarks, as this two sectors become independent in the infinite mass limit. The lowest lying pseudoscalar and vector heavy mesons, relevant to this work, belong to a multiplet under the I-W symmetry, and must therefore be treated together. Their masses are equal, up to symmetry breaking corrections of hyperfine origin equal to Λ^2/m_Q (Λ is a typical QCD scale, in this specific case $\Lambda \sim m_\rho/\sqrt{2}$) , and, their transition amplitudes become related. For all amplitudes where the relevant scale is Λ, the heavy quark velocity is conserved. These conservation laws obviously extend to the heavy-meson Goldstone-boson interactions relevant in the present context. For this reason, it is convenient to consider as the starting point the effective QCD theory for the heavy quark sector, defined in terms of heavy quarks and antiquarks separately (i.e., in the effective theory no virtual loops of heavy quarks are required) for each 4-velocity v_μ [2]. The corresponding effective theory for the heavy (anti)-mesons is obtained in a similar way, by defining non-relativistic fields for mesons ($\overline{Q}q$) and anti-mesons ($\bar{q}Q$) for a given 4-velocity v_μ as follows:

$$\begin{aligned} H_v^{(+)}(x) &= \sqrt{M_H}\, e^{iM_H v\cdot x}\, \Psi_+(x) \\ H_v^{(-)}(x) &= \sqrt{M_H}\, e^{iM_H v\cdot x}\, \Psi_-^\dagger(x) \end{aligned} \quad , \qquad (6)$$

where $\Psi_{+,-}(x)$ are the positive and negative frequency components of the relativistic meson field $\Psi(x)$ and M_H is the heavy meson mass. Since the meson and antimeson sectors become independent, we will only work with mesons, and the field $H_v^{(+)}(x)$, which annihilates heavy mesons with velocity v_μ, will be simply denoted by $H(x)$.

For heavy mesons, the I-W symmetry is elegantly implemented by merging the pseudoscalar and vector mesons into a multiplet as follows [13, 14]:

$$\mathcal{H}(x) = \frac{1+\not{v}}{2}\left(-\gamma_5 H(x) + \gamma_\mu H^\mu(x)\right) \quad , \qquad (7)$$

where the vector field H^μ satisfies the constraint $v_\mu H^\mu = 0$. For further convenience, the field conjugated to $\mathcal{H}$ which will create heavy mesons is defined by $\overline{\mathcal{H}} = \gamma_0 \mathcal{H}^\dagger \gamma_0$. Under chiral rotations $\mathcal{H}$ transforms as indicated in

(1), and under the heavy quark symmetry rotations its transformation law is the following:

$$\begin{aligned} \mathcal{H} &\rightarrow e^{i\theta_j S_j}\, \mathcal{H} \\ S_j &= i\,\epsilon_{jkl}\,[\ \not{e}_k\ ,\ \not{e}_l\ ] \end{aligned} \quad , \qquad (8)$$

where e_i^μ, $i = 1,2,3$ are normalized space-like vectors orthogonal to v_μ.

Since the definition (6) corresponds in the rest frame of the meson to the subtraction of the rest mass energy, the operator $-i\partial_\mu$ acting on $\mathcal{H}$ gives the residual momentum. In particular, for the purposes of the chiral expansion, this residual momentum will count as a quantity of $\mathcal{O}(p)$. Similarly, $\partial_\mu u$ is of $\mathcal{O}(p)$, and, therefore, the covariant derivative only contains terms of $\mathcal{O}(p)$ as is also the case for ω_μ.

It is now straightforward to write down the lowest order effective Lagrangian, in both chiral and $1/m_Q$ expansions, which is invariant under chiral and I-W transformations as well as under parity and charge conjugation. This Lagrangian is of $\mathcal{O}(p)$ and reads as follows:

$$\begin{aligned} \mathcal{L}^{(1)} &= -\frac{1}{2}\, v_\mu\, Tr_D\, \{\, \overline{\mathcal{H}} \nabla^\mu \mathcal{H}\, \} \\ &+ \frac{1}{2}\, g\, Tr_D\, \{\, \overline{\mathcal{H}}\, \omega^\mu\, \mathcal{H}\, \gamma_\mu \gamma_5\, \} \end{aligned} \quad , \qquad (9)$$

where Tr_D denotes the trace over Dirac indices. The first term contains the kinetic energy and interactions of the heavy mesons with an even number of Goldstone bosons and no change in the spin of the heavy meson. This term is universal and automatically satisfies the I-W symmetry. The second term gives rise to virtual transitions $H^* + m\pi \leftrightarrow H + n\pi$ and $H^* + m\pi \leftrightarrow H^* + n\pi$ with $(n+m)$ odd. The I-W symmetry imposes that the strength of both types of transitions must be equal. The corresponding adimensional coupling constant g could be determined from the decay $D^{*+} \rightarrow D + \pi$; unfortunately, at present, only an upper bound on the D^{*+} width is available, resulting in $g^2 \leq 4.8$. One may take as a rough estimate for g^2 the corresponding coupling constant in the K meson system: $g^2_{K^*K\pi} = 0.46$. The quark model result is $g^2 \sim 0.3$ [15], while the QCD sum-rules give the substantially smaller value $g^2 \sim 0.08$ [16].

Notice that $\mathcal{L}^{(1)}$ does not contain the heavy meson masses. This is the key point in the implementation of the power counting of the low energy expansion as a loop expansion [6]. The Feynman rules are straightforward to derive and the propagators for the heavy mesons are given by:

$$\begin{aligned} \Delta_H(p) &= \frac{i}{2\,p.v + i\epsilon} \\ \Delta^{\mu\nu}_{H^*}(p) &= -i\,\frac{(g^{\mu\nu} - v^\mu v^\nu)}{2\,p.v + i\epsilon} \qquad , \end{aligned} \tag{10}$$

where p_μ is the residual momentum. The use of these propagators in chiral loop integrals is justified, since the physical cutoff for such integrals is $\Lambda = \mathcal{O}(1GeV)$ (actually, since in the effective theory we integrate out resonances, e.g. D_1, the cutoff should be smaller than the mass difference between the resonances and the stable states). The implied change in the U.V. degree of divergence of the integrals, which occurs at a scale of momenta of the order of the heavy meson mass, is therefore irrelevant. In calculating the chiral loops it is convenient to use dimensional regularization, as it preserves chiral invariance. Having eliminated the heavy meson mass, only low energy scales appear in the loops, thus furnishing the chiral power counting as in the case of light mesons [7].

The most noticeable effect of $SU(3)$ breaking by the quark masses is in the masses of the heavy mesons. The leading contribution to the intramultiplet mass splittings is linear in the light quark masses (which in chiral power counting are of $\mathcal{O}(p^2)$) and can be described by adding the following $\mathcal{O}(p^2)$ term to the effective Lagrangian:

$$\mathcal{L}^{(2)}_{\Delta M} = -\frac{C}{4}\, Tr_D\,\{\,\overline{\mathcal{H}}\,(u^\dagger \mathcal{M} u^\dagger + u\mathcal{M}u)\,\mathcal{H}\,\} \qquad , \tag{11}$$

where $\mathcal{M} = \mathrm{diag}(\,m_u, m_d, m_s\,)$ is the light quark mass matrix[#] . The $SU(3)$ singlet term has been omitted as it is of no interest for our purposes. Under the assumption that higher chiral order corrections are small (more on this in the next section) and neglecting them, C is estimated from $M_{D_s} - M_{D^+}$ and $M_{B_s} - M_{B^0}$ and the results are: $C_D = 99.5 \pm 0.6 MeV/(m_s - m_d)$, $C_B = (82 \pm 2.5; 121 \pm 10) MeV/(m_s - m_d)$. For $M_{B_s} - M_{B^0}$ we use the two values quoted as consistent with present data [17]. Establishing this measurement would provide an estimate of the $1/m_Q$ corrections by comparison of C_D with C_B. Notice that $C_K \sim 225 MeV/(m_s - m_d)$, as obtained from isospin breaking in the kaon masses and using the ratio $R = (m_s - \hat{m})/(m_d - m_u)$ ($\hat{m} = (m_u + m_d)/2$), which is substantially larger than in heavy mesons. A re-analysis of isospin breaking effects within the linear approximation has been recently done [18]. The leading corrections to the linear approximation are non-analytic in the light quark masses $\sim m^{3/2}$, and they turn out to be proportional to g^2, as we will show in next section. The possibility that these corrections turn out to be important is not yet excluded.

3 Decay Constants and Masses

In heavy mesons, as in light mesons, the leading $SU(3)$ breaking correction to decay constants is proportional to the light quark masses multiplied by a non-analytic factor, the chiral logarithm, which emerges due to the I.R. behaviour of the one chiral loop integrals.

The decay constants are defined in terms of matrix elements of the vector and axial vector currents $V^i_\mu = \overline{Q}\gamma_\mu q^i$ and $A^i_\mu = \overline{Q}\gamma_\mu\gamma_5 q^i$ between one meson state and the vacuum. In the effective theory, these currents are defined by introducing vector (v_μ) and pseudovector (a_μ) external sources which are

The form of counterterms containing the quark masses is determined by implementing full chiral invariance replacing the quark mass matrix by sources $(s + ip)$ [7].

triplets under $SU(3)$. These sources couple to the mesons at lowest chiral order ($\mathcal{O}(p)$) according to the following effective Lagrangian:

$$\mathcal{L}^{(1)}_{\rm source} = \frac{1}{2}\, f\, Tr_D\,\{\left(v^{\dagger}_{\mu} + a^{\dagger}_{\mu}\gamma_5\right)\ \gamma_{\mu}\ \left[(u+u^{\dagger}) + \gamma_5(u-u^{\dagger})\right]\mathcal{H}\} + \text{h.c.} \quad (12)$$

Clearly, the sources defined here in momentum representation will only carry residual momentum, and, therefore, they count as quantities of $\mathcal{O}(p)$. f is defined in the $m_Q \to \infty$ and chiral limits and related to the usual expressions for the decay constants of the pseudoscalar and heavy mesons in that limit by:

$$\begin{aligned} F_H &= f/\sqrt{M_H} \\ F_{H^*} &= f\sqrt{M_H} = F_H M_H \end{aligned} \quad (13)$$

The leading $SU(3)$ breaking corrections to the ratios of decay constants and the leading non-analytic (in the light quark masses) corrections to the mass differences are determined by calculating the two polarizations $\Pi^{A\,ij}_{\mu\nu}(x) = \langle 0 \mid T A^i_{\mu}(x) A^j_{\nu}(0) \mid 0\rangle$ and $\Pi^{V\,ij}_{\mu\nu}(x) = \langle 0 \mid T V^i_{\mu}(x) V^j_{\nu}(0) \mid 0\rangle$ to one loop order. At long Euclidean distances, the lowest lying pseudoscalar and vector heavy meson poles respectively saturate the two point functions. These pole contributions are given in the effective theory by replacing the currents by the effective currents derived from the source Lagrangian (12). The corresponding Fourier transforms, which are functions of the residual momentum p_{μ}, are given in the limit $p.v \to 0$ by the following general expressions:

$$\begin{aligned} \tilde{\Pi}^{A\,ij}_{\mu\nu}(p) &= \frac{4}{M_H} F^2_{H_i}\, v_{\mu}v_{\nu}\, \frac{i}{2\,(p.v - \delta M_i) + i\epsilon}\,\delta_{ij} \\ \tilde{\Pi}^{V\,ij}_{\mu\nu}(p) &= 4 M_H F^2_{H^*_i}\, \frac{-i\,(g_{\mu\nu} - v_{\mu}v_{\nu})}{2\,(p.v - \delta M_i) + i\epsilon}\,\delta_{ij} \end{aligned} \quad (14)$$

The diagrams contributing to one chiral loop order are shown in fig. I. Since these contributions are U.V. divergent, counterterms are required. It turns out that one only needs to add counterterms which correct the effective

currents and which are of $\mathcal{O}(p^3)$. Since, as expected, the chiral loops turn out to preserve the heavy quark symmetry, the counterterms must be invariant under this symmetry as well. The counterterm effective Lagrangian contains three low energy constants, and is given by:

$$\begin{aligned} \mathcal{L}^{(3)}_{\rm source} &= 2B_0\,\frac{f}{F_0^2}\times \\ & \Big\{\Gamma_1\, Tr_D\,\{\left(v^{\dagger}_{\mu} + a^{\dagger}_{\mu}\gamma_5\right)\gamma^{\mu}\left[(U\mathcal{M}u + U^{\dagger}\mathcal{M}u^{\dagger}) + \gamma_5(U\mathcal{M}u - U^{\dagger}\mathcal{M}u^{\dagger})\right]\mathcal{H}\} \\ &+\ \Gamma_2\, Tr_D\,\{\left(v^{\dagger}_{\mu} + a^{\dagger}_{\mu}\gamma_5\right)\gamma^{\mu}\left[(\mathcal{M}u + \mathcal{M}u^{\dagger}) + \gamma_5(\mathcal{M}u^{\dagger} - \mathcal{M}u)\right]\mathcal{H}\} \\ &+\ \Gamma_3\, Tr\{\mathcal{M}U^{\dagger} + \mathcal{M}^{\dagger}U\} Tr_D\,\{\left(v^{\dagger}_{\mu} + a^{\dagger}_{\mu}\gamma_5\right)\gamma^{\mu}\left[(u+u^{\dagger}) + \gamma_5(u-u^{\dagger})\right]\mathcal{H}\}\Big\} \\ &+\ \text{h.c.} \quad , \end{aligned} \quad (15)$$

where $B_0 = \langle\bar{q}q\rangle_0/F_0^2$ is defined in the chiral limit ($M_{\pi}^2 = B_0(m_u + m_d)$, etc.).

We calculate the loops using dimensional regularization, in which it is convenient to write: $\Gamma_j = \Gamma^r_j(\mu) + \overline{\Gamma}_j\lambda(\mu)$, ($j = 1,2,3$), where μ is the chiral renormalization scale, $\Gamma^r_j(\mu)$ the renormalized effective coupling, and $\lambda(\mu)$ contains the singularity at $d = 4$ and is given by:

$$\lambda(\mu) = \frac{1}{16\pi^2}\mu^{d-4}\{\frac{1}{(d-4)} - \frac{1}{2}(\log 4\pi + \Gamma'(1) + 1)\} \quad (16)$$

The following choice leads to an U.V. finite result for the polarizations:

$$\begin{aligned} \overline{\Gamma}_1 + \overline{\Gamma}_2 &= \frac{5}{24}\,\left(1 + 3g^2\right) \\ \overline{\Gamma}_3 &= \frac{11}{144}\,\left(1 + 3g^2\right) \end{aligned} \quad (17)$$

For the sake of convenience we define: $\Gamma^r_{12}(\mu) \equiv \Gamma^r_1(\mu) + \Gamma^r_2(\mu)$.

We first consider $\tilde{\Pi}^{A\,ij}_{\mu\nu}$. One loop contributions from $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(1)}_{\rm source}$ and a tree-level insertion of $\mathcal{L}^{(2)}_{\Delta M}$ must be included, as shown in fig. I (a). The calculation is straightforward, and leads to the following results for the pseudoscalar decay constants in the $SU(2)$-limit:

$$F_{H_{u,d}} = F_H\,\{1 - \left(\frac{1+3g^2}{8F_0^2}\right)[3\mu_{\pi} + 2\mu_K + \frac{1}{3}\mu_{\eta}]$$

$$+ \frac{1}{F_0^2}\,\Gamma^r_{12}(\mu)\,2M_\pi^2 + \frac{4}{F_0^2}\,\Gamma^r_3(\mu)\,\left(2M_K^2 + M_\pi^2\right)\}$$

$$F_{H_s} = F_H\,\{1 - \left(\frac{1+3g^2}{8F_0^2}\right)[4\mu_K + \frac{4}{3}\mu_\eta] \qquad (18)$$

$$+ \frac{2}{F_0^2}\,\Gamma^r_{12}(\mu)\,(2M_K^2 - M_\pi^2) + \frac{4}{F_0^2}\,\Gamma^r_3(\mu)\,(2M_K^2 + M_\pi^2)\} \quad ,$$

where the chiral logarithms are given by:

$$\mu_P = M_P^2 \frac{1}{16\pi^2} \log \frac{M_P^2}{\mu^2} \qquad (19)$$

From these expressions one easily finds the $SU(3)$ breaking corrections to the ratio $F_{H_s}/F_{H_{u,d}}$:

$$\frac{F_{H_s}}{F_{H_{u,d}}} = 1 - \left(\frac{1+3g^2}{8F_0^2}\right)[-3\mu_\pi + 2\mu_K + \mu_\eta] + \frac{4}{F_0^2}\,\Gamma^r_{12}(\mu)\,(M_K^2 - M_\pi^2) \qquad (20)$$

This result, which coincides with that of ref [10], implies that the decay constant grows with the mass of the light quark in the chiral limit, as it also occurs in light mesons. In the case of light mesons, the correction to F_K/F_π is entirely contained in the chiral logarithm term if one takes $\mu \sim 1.5\ GeV$ [7]. If a similar situation is assumed to hold in the case of the heavy mesons, one obtains $F_{H_s}/F_{H_{u,d}} - 1 \sim 0.13\,(1+3g^2)$ (clearly, this should only be taken as a rough estimate of the size of the effect). A correction of this size should be experimentally accessible in the future in leptonic D-meson decays. A similar direct test is not available for B mesons because B_s is neutral.

The mass shifts δM_i receive an $\mathcal{O}(p^2)$ contribution from the insertion of $\mathcal{L}^{(2)}_{\Delta M}$, and a non-analytic contribution in the quark masses of $\mathcal{O}(p^3)$ from the loop diagram proportional to g^2. The latter is similar to the non-analytic contribution to the mass splittings in the baryon octet identified long ago [19]. The non-analyticity here, as in the case of the chiral logarithms, is a long distance effect due to the I.R. behaviour of the chiral loop integral. Since local counterterms must be analytic in the quark masses, these non-analytic corrections are unambiguous, as one would expect from their long

distance nature (for details see [20]). It is interesting to note that they do not depend on the fact that we are using the leading term in the $1/m_Q$ expansion within the loop integral; exactly the same result is obtained by doing the loop integral in the relativistic theory, and expanding the result. In the $SU(2)$ limit, we obtain the following expression for the mass splitting:

$$\delta M_{H_s} - \delta M_{H_{u,d}}|_{QCD} = C(m_s - \hat{m}) - \frac{3g^2}{128\pi F_0^2}[-3M_\pi^3 + 2M_K^3 + M_\eta^3]$$
$$\hat{m} = m_u = m_d \qquad (21)$$

Notice that the non-analytic term has the opposite sign from the leading term and gives a large contribution, unless the effective coupling g^2 is very small. For instance, for its contribution to be less than 20%, $g^2 < 0.06$ is required. For this reason, it is important at some point to determine this coupling constant with a good degree of confidence, since, among other effects, it could lead to sizeable corrections to the linear approximation normally used in the analysis of isospin breaking in heavy mesons. In particular, $C_{D,B}$ will become closer to C_K. A few remarks are in order here: a- Due to $SU(3)$ breaking the propagators can not be brought back to the form (10) because chiral invariance demands that only $SU(3)$ singlet redefinitions of the form (6) are admissible. Thus, $SU(3)$ breaking implies that propagators will in general contain an $\mathcal{O}(p^2)$ residual mass. b- The propagator of the heavy meson in the chiral loop could have been taken with the $\mathcal{O}(p^2)$ corrections given by the insertion of $\mathcal{L}^{(2)}_{\Delta M}$. This, however, only produces a correction of $\mathcal{O}(p^4)$ while it does not affect the leading non-analytic term. c- Clearly, some corrections step by one unit in the chiral power counting, making predictions more difficult as they are only suppressed by a factor $\sim M_K/4\pi F_0$ which is not much smaller than one. For instance, it could well occur that a large leading non-analytic term such as that in (21) becomes partly compensated by a term of $\mathcal{O}(p^4)$. In this particular case, an estimate of the $\mathcal{O}(p^4)$ chiral logarithm shows that its contribution will be small if one chooses $\mu = \mathcal{O}(1GeV)$, however, the $\mathcal{O}(p^4)$ counterterm could eventually lead to the mentioned compensation. If g^2 turns out to be substantially larger than 0.06, one would then have an indication for such a compensation.

The relevant one loop contributions to $\tilde{\Pi}^{V\,ij}_{\mu\nu}(p)$ are shown in fig. I (b), and determine the corrections to the masses and decay constants of the vector mesons. Explicit calculation shows that the heavy quark symmetry is preserved, as seen in particular by the relation $F_{H^*} = F_H M_H$, which still holds after the chiral loop corrections are included.

One might wonder about the precision of the chiral loop corrections in the heavy quark limit when applied to D and B mesons. Calculations of F_D and F_B in lattice QCD [21] and QCD sum rules [22] have shown that the $1/\sqrt{m_Q}$ scaling characteristic of the heavy quark limit is not present. On the other hand, there is clear evidence that the scaling violation mainly stems from spin independent effects, and therefore that the heavy quark-symmetry breaking effects on the ratio $F_{H^*}/F_H M_H$ are small: ($\sim 10\%$) for the B mesons [23] and somewhat larger for D mesons ($\sim 20\%$). Analogously, the deviation from unity of the ratio of effective couplings $g_{HH^*\pi}/g_{H^*H^*\pi}$ is expected to be small, since it is also of hyperfine origin. This deviation and the vector-pseudoscalar mass splitting are the main source of departure of the chiral corrections from the $m_Q \to \infty$ limit. We expect, therefore, that for D and B mesons their departure from this limit will be small (this involves the reasonable assumption that also the $1/m_Q$ corrections to the counterterms will be small).

4 $B \to \overline{D}^{(*)}$ Form Factor

In the infinite mass limit, the heavy quark symmetry permits one to determine the amplitudes for $B \to \overline{D}^{(*)}$ transitions mediated by the charged currents $\bar{c}\gamma_\mu(\gamma_5)b$ in terms of a single real form factor $\xi(v.v')$ [3], where v (v') is the 4-velocity of the b (c) quark. At the vanishing recoil point, ξ satisfies the normalization condition $\xi(v.v' = 1) = 1$.

In this section, we will determine the form of the leading $SU(3)$ breaking corrections to this form factor. To leading order in the chiral and $1/m_Q$ expansions the effective charged currents are obtained from the following source Lagrangian:

$$\mathcal{L}^{(1)}_{b\to c} = \xi(\nu)\, Tr_D\, \{\overline{\mathcal{D}}\, \mathcal{B}\ (V^\mu + A^\mu\gamma_5)\,\gamma_\mu\} + \text{h.c.} \qquad , \tag{22}$$

where $\mathcal{D}$ and $\mathcal{B}$ are the expressions analogous to (7) for D and B mesons respectively, $\nu = v.v'$, and V^μ and A^μ are $SU(3)$ singlet sources. In particular, (22) shows that the effective currents do not couple to the Goldstone modes at lowest chiral order.

The chiral corrections to the matrix element $\langle \overline{D}_j \mid \bar{c}\gamma_\mu b \mid B_i\rangle$ are calculated by considering the three-point function
$\Pi^{V\,i,j}_{\rho\mu\nu}(x,y) = \langle 0 \mid T\ \bar{c}\gamma_\rho b(0)\ \ \bar{q}_j\gamma_\nu\gamma_5 c(y)\ \ \bar{b}\gamma_\mu\gamma_5 q_i(x) \mid 0\rangle$. In the effective meson theory, the corresponding three-point function in the residual momentum representation and in the limit $p.v \sim p'.v' \sim 0$, where p_μ (p'_μ) is the residual momentum associated with the B (D) meson propagator, has the following form:

$$\begin{aligned} \tilde{\Pi}^{V\,i,j}_{\rho\mu\nu}(p,v,p',v') \quad &= \\ -\frac{4}{\sqrt{M_B M_D}}\, F_{B_i} F_{D_j}\ \ v_\mu v'_\nu\ \ (v+v')_\rho\, \xi_{ij}(\nu)\, &\frac{i}{2\,(p.v - \delta M_i)}\, \frac{i}{2\,(p'.v' - \delta M_j)} \end{aligned} \tag{23}$$

To one chiral loop order, the diagram shown in fig. II (a) gives the correction to the I-W form factor, after properly taking into account wave function renormalization (the latter naturally emerges when explicitly calculating all one loop diagrams for the three-point function). The result for the form factor is as follows:

$$\xi_{ij}(\nu) = \xi(\nu)\,(\delta_{ij} + \omega_{ij}(\nu)) \tag{24}$$

where $\omega_{ij}(\nu)$ is given by the following expression:

$$\omega_{ij}(\nu) \quad = \quad \Omega(\nu)\,\Delta_{ij} + \text{counterterm}$$

$$\begin{aligned}
\Delta_{ij} &= \frac{1}{F_0^2}\sum_{a=1}^{8}(\lambda_a^2)_{ij}\left(M_a^2\lambda+\frac{1}{2}\mu_a\right)\\
\Omega(\nu) &= g^2\left(-\frac{3}{2}+(2+\nu)\,A(\nu)+B(\nu)\right) \qquad (25)\\
A(\nu) &= \frac{1}{2\sqrt{\nu^2-1}}\,\log\left(\frac{\nu+1+\sqrt{\nu^2-1}}{\nu+1-\sqrt{\nu^2-1}}\right)\\
B(\nu) &= \frac{1}{2}+\frac{\nu}{4\sqrt{\nu^2-1}}\,\log\left(\frac{\nu-\sqrt{\nu^2-1}}{\nu+\sqrt{\nu^2-1}}\right)
\end{aligned}$$

Clearly, Δ_{ij} is diagonal. Expanding at zero recoil ($\nu=1$), the first few terms are as follows:

$$\Omega(\nu)=g^2\left(-\frac{1}{3}(\nu-1)+\frac{2}{15}(\nu-1)^2-\frac{2}{35}(\nu-1)^3+\cdots\right) \qquad (26)$$

As expected from the fact that the effective vector current is conserved at leading order in $1/m_Q$ as consequence of the I-W symmetry (more specifically, the part of the symmetry which corresponds to the flavor rotation between c and b quarks), the corrections vanish at zero recoil. The counterterms needed to render results U.V. finite only affect the definition of the effective current. They are of $\mathcal{O}(p^2)$ and given by:

$$\begin{aligned}
\mathcal{L}^{(2)}_{b\to c} &= 2B_0\,\xi(\nu)\,\frac{\Omega(\nu)}{F_0^2}\times \qquad (27)\\
&\left\{\eta_1(\nu)\,Tr_D\left\{\overline{\mathcal{D}}(u\mathcal{M}u+u^\dagger\mathcal{M}u^\dagger)\mathcal{B}(\mathrm{V}_\mu+\mathrm{A}_\mu\gamma_5)\gamma^\mu\right\}\right.\\
&+\left.\eta_2(\nu)\,Tr(\mathcal{M}U^\dagger+\mathcal{M}U)\,Tr_D\,\mathcal{B}\,(\mathrm{V}_\mu+\mathrm{A}_\mu\gamma_5)\gamma^\mu\}\right\}
\end{aligned}$$

where the following choice of effective couplings provides a finite result for the three-point function:

$$\begin{aligned}
\eta_1(\nu) &= \eta_1^r(\nu;\mu)-\frac{5}{6}\,\lambda(\mu) \qquad (28)\\
\eta_2(\nu) &= \eta_2^r(\nu;\mu)-\frac{11}{18}\,\lambda(\mu)
\end{aligned}$$

Notice that, in accordance with the normalization condition, the counterterms also have to vanish at zero recoil.

After replacing in ω_{ij} in (25) the contribution of the counterterms, one finds the following $SU(3)$ breaking correction to the ratio of form factors:

$$\frac{\xi_s(\nu)}{\xi_{u,d}(\nu)}=1+\frac{\Omega(\nu)}{F_0^2}\left(\mu_K+\frac{1}{2}\mu_\eta-\frac{3}{2}\mu_\pi+4\,\eta_1^r(\nu;\mu)\,(M_K^2-M_\pi^2)\right) \qquad (29)$$

This result shows that for small quark masses the chiral logarithm tends to increase the value of ξ_s with respect to $\xi_{u,d}$. This result seems to be anti-intuitive; one would expect that the heavier the light quark is the faster the form factor will drop with increasing ν. This is certainly true for large enough light quark mass. As the chiral limit is approached, however, the behavior reverses. Such a behaviour is known to occur, for instance, in the quark-antiquark condensates, which, near the chiral limit increase with the quark mass, yet start decreasing as the mass becomes large enough. Notice that the form of the correction is algebraically of the same form as the one for the ratio of decay constants. The explicit ν-dependence of η_{12} indicates that the counterterm can change the profile of the correction with respect to that given by $\Omega(\nu)$.

It is important to emphasize that the chiral loop result (24, 25) holds for any value of the recoil. The behavior of $\Omega(\nu)$ is smooth, and for $\nu\to\infty$ it tends to $-g^2$. In particular, it applies to the whole Dalitz domain of the semileptonic decays $B\to\overline{D}l\nu_l$, and to non-leptonic decays in the factorization limit (e.g. $B^0\to D^+\pi^-$ and $B_s\to\overline{D}_s\pi^-$). At the largest available recoil $\nu\sim 1.8$, we have $\Omega\sim -0.2\,g^2$. Choosing values for μ between 1 and 1.5 GeV, the correction to the the ratio (29) contributed by the chiral logarithms is $\sim -\Omega(\nu)\times(0.3$ to $0.5)$. This implies that even at the largest recoil this contribution will be small, perhaps, only a few percent. Thus, unless the counterterm contribution is surprisingly large, the $SU(3)$ breaking effects on the form factor will be in the few percent range. Notice that the size of this correction and the non-analytic contributions to the mass splittings are related, since both are proportional to g^2.

As in the case of the decay constants, we explicitly checked that the heavy quark symmetry is preserved by the chiral loops. This check was done by considering the three-point function

$\Pi^{A\,i,j}_{\rho\mu\nu}(x,y) = \langle 0 \mid T\ \bar{c}\gamma_\rho\gamma_5 b(0)\ \ \bar{q}_j\gamma_\nu c(y)\ \ \bar{b}\gamma_\mu q_i(x) \mid 0\rangle$. In this case, the corrections to the form factor are obtained from the diagrams in fig. II (b).

Finally, $SU(3)$ breaking gives rise to direct couplings of the Goldstone bosons to the effective currents, as shown by the counterterm (27). They are proportional to the light quark masses, and therefore small and unlikely to be of direct physical relevance.

5 Remarks

It is certainly worthwhile to explore possible effects of chiral symmetry in the physics of heavy hadrons. It is likely that they will mainly be restricted to the light quark mass induced $SU(3)$ breaking corrections as are the ones discussed in this work. Less clear is the accessibility to the predictions of low energy theorems for soft meson emission in decays. At any rate, substantial experimental improvement, especially in strange heavy mesons, is required until effects beyond the mass splittings are accessible. As the predictive power of the chiral expansion is limited by the need of introducing counterterms (as we saw in the cases of the decay constants and the I-W form factor), it is not clear that enough experimental information will become available as reach the stage of testing these limited predictions.

We expect F_{D_s}/F_D, for which the $SU(3)$ breaking correction might be of the order of $10 - 20$ %, to be a first candidate to be measured. The corrections to the I-W form factor will be much harder to observe, since this will require large numbers of B_s mesons, and the effect itself might be very small. The situation could be improved by considering some semi-inclusive

decays where the chiral expansion is well defined, for instance, channels for which factorization holds to a good degree. The leading non-analytic corrections to the mass splittings might be large, depending on the value of g^2, and could, therefore, affect analyses on isospin breaking done within the linear approximation.

Lattice QCD simulations of heavy hadrons might also be an interesting domain of application. For example, chiral symmetry controls the finite volume effects, which are fully predictable for the unquenched theory. The problem of determining the effective couplings, like those appearing in the counterterms $\mathcal{L}^{(3)}_{source}$ and $\mathcal{L}^{(2)}_{b\to c}$ might well be first solved on the lattice by looking at the light quark-mass dependence of the observables we discussed.

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Note added: after submition for publication of this work, the authors of ref. [24] made us aware of their calculation of the chiral corrections to the $B \to \overline{D}^{(*)}$ form factor.

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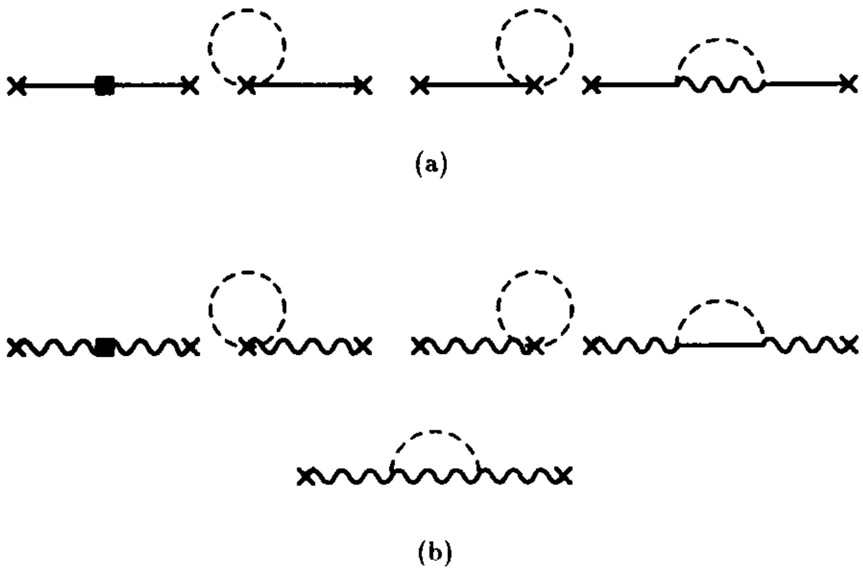

Fig. I : One loop contributions to the polarizations a) $\tilde{\Pi}^{A\,ij}_{\mu\nu}$ and b) $\tilde{\Pi}^{V\,ij}_{\mu\nu}$. The solid lines correspond to the heavy pseudoscalar, the wavy line to the heavy vector and the dashed line to the Goldstone bosons. The square dot represents the insertion of $\mathcal{L}^{(2)}_{\Delta M}$. Diagrams not explicitly shown vanish identically.

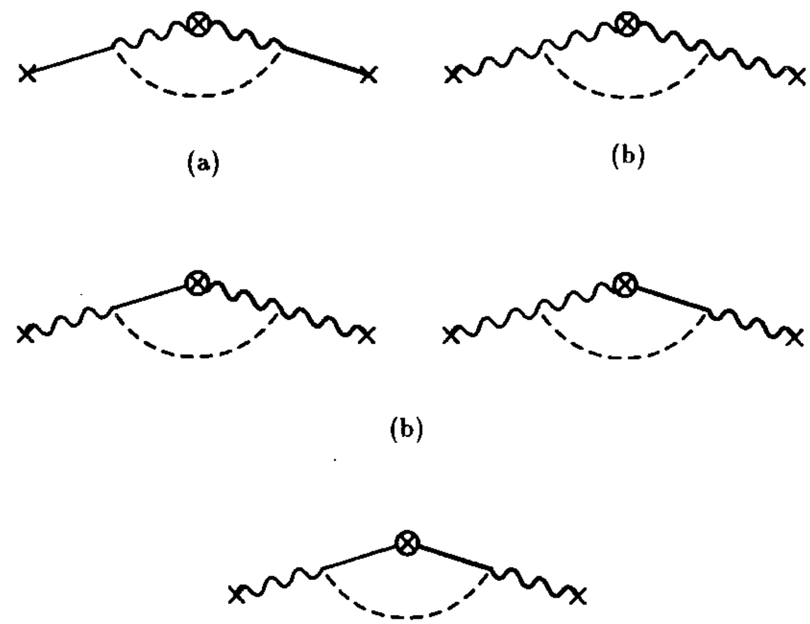

Fig. II : One loop correction to the form factor $\xi(v.v')$ as determined from the three point functions a) $\tilde{\Pi}^{V\,i,j}_{\rho\mu\nu}$ and b) $\tilde{\Pi}^{A\,i,j}_{\rho\mu\nu}$. Diagrams which only contribute to decay constants, masses and wave function renormalization are not displayed.